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TEMPERATURE TRANSITIONAL PHENOMENA IN SPHERICAL RESERVOIR WALL

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Original scientific paper

Uneven and non-stationary temperature field is formed in the reservoir wall while loading pressure reservoir with a liquefied gas in cooling state, at a temperature lower than the saturation temperature at given pressure. Mathematical model of the temperature field is developed by setting and solving differential equation of heat diffusion in the spherical wall in real conditions of the wall getting cold. The model enables computation of the temperature gradient attaining very high values at the liquid level. A simplified procedure for calculating the wall temperature increase in the narrow area above the liquid level was also developed. Understanding of the temperature field in the reservoir wall allows further heat stresses calculation. For this purpose, a temperature discontinuity should be approximated by a continuous function. Approximation was applied by Fourier's orders, and coefficients of an order are determined by the electronic computer program. The model application procedure has been illustrated by the example of spherical pressure reservoir for liquefied carbon dioxide.

Keywords: *liquefied gas, spherical reservoir, temperature field*

Temperaturne prijelazne pojave u stijenci sfernog spremnika

Izvorni znanstveni članak

Tijekom punjenja tlačnog spremnika ukapljenim plinom, pothlađenim ispod temperature zasićenja za zadani tlak, nastaje u stijenci spremnika nejednoliko i nestacionarno temperaturno polje. Matematički model temperaturnog polja razvijen je postavljanjem i rješavanjem diferencijalne jednadžbe provođenja topline u sfernoj stijenci u realnim uvjetima ohlađivanja stijenke. Model omogućava izračunavanje temperaturnog gradijenta koji na visini razine kapljevine poprima vrlo visoke vrijednosti. Razvijen je također i pojednostavljeni postupak za izračunavanje iznosa porasta temperature stijenke u uskom području iznad razine kapljevine. Poznavanje temperaturnog polja u stijenci spremnika omogućava dalje izračunavanje toplinskih naprezanja. U tu svrhu diskontinuitet temperature treba aproksimirati kontinuiranom funkcijom. Primijenjena je aproksimacija pomoću Fourierovih redova, a koeficijenti reda se određuju pomoću programa za elektroničko računalo. Postupak primjene modela ilustriran je na primjeru sfernog tlačnog spremnika za ukapljeni ugljični dioksid.

Ključne riječi: *sferni spremnik, temperaturno polje, ukapljeni plin*

1 Introduction
Uvod

Spherical vessels are widely used as pressure reservoirs for liquefied petroleum gas (LPG), carbon dioxide, ammonium, and many other technical gasses. Operating pressure and temperatures the reservoir walls are exposed to depend on the stored gas type i.e. its saturated-liquid curve parameters. In some types gasses are stored at an environment temperature at atmospheric or increased pressure. For example, calculated temperature in designing pressure vessels for LPG is regulated at 40 °C by the Croatian standards. Associated saturation pressure i.e. the lowest pressure at which the gas is allowed to convert into a liquid state at a given temperature is accepted as a calculated pressure. The other way is storing gas in cooling conditions i.e. at temperatures lower than the environment ones. For example, carbon dioxide has, at environment temperatures, high saturation pressures (at 25 °C $\rightarrow$ 64,3 bars). This gas can be obtained in a liquefied state even at lower pressures, if kept in a cool state (at –25 °C $\rightarrow$ 16,8 bars), but not lower than 5,18 bars since then sublimation occurs. The cold state is desirable to be maintained with ammonia although its storing in a liquefied state, at an environment temperature, does not require too high pressures (at 25 °C $\rightarrow$ 10 bars). Namely, ammonia is very dangerous for health condition, thus, protection measures from unexpected outflow and spreading into atmosphere are very important. Thus, accident danger is considerably lower if cold ammonia is kept at atmospheric pressure, liquefied at –31°C.

In certain operating conditions the vessels are exposed to significant temperature change effects. Temperature field gradient can, at some points of the vessel wall, reach high values due to the effect of non-homogeneous and non-stationary temperature field. Stresses and deformations caused by this gradient have dynamic character in the transferable period and may significantly change original image of the stress field. In order to analyse temperature stresses in the vessel walls it is necessary to anticipate critical temperature distribution per thickness and per wall surface as well as dynamics of its change in the vessel designing phase. It means that non-stationary temperature field, expected to occur in real conditions of the vessel exploitation, should be modelled.

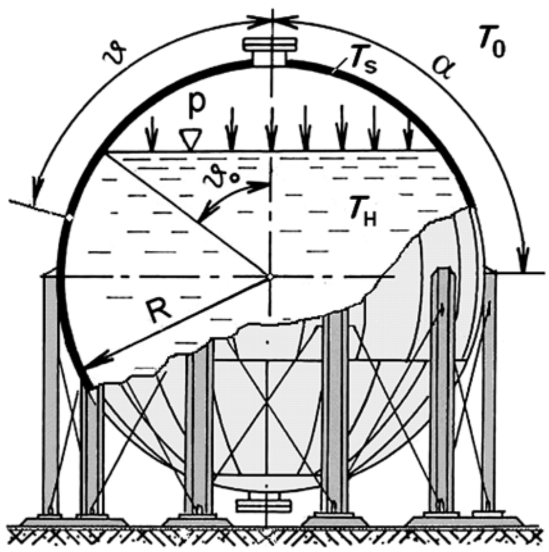

Figure 1 *Spherical shape pressure reservoir for liquefied gas*
Slika 1. *Tlačni spremnik sfernog oblika za ukapljeni plin*

Generally, especially while talking about fast temperature changes or high speeds of deforming, both temperature field and the deformation field are coupled quantities. Thus, mathematical methods of their determination are very complex and are studied in a coupled dynamic theory of thermo elasticity [1]. Such approach is necessary in studying the phenomena of thermo-elastic dissipation of energy. Since elastic body deformation causes slight change of its temperature, mutual action effect of the temperature and deformation field is not significant for problems of temperature stresses determination in common conditions. In this case the problem can be reduced to quasi-static task of thermo-elasticity where non-stationary temperature field is considered not coupled with shifts field. In this way temperature field can be defined prior to determination of deformation field.

2. Modelling of a temperature field
Modeliranje temperaturnog polja

The topic of the consideration will be the problem of modelling the temperature field occurring in the spherical reservoir wall in the course of loading the reservoir with liquefied gas. It is characterized by considerably lower temperature than the one of the reservoir wall. The vessel wall is supposed to be heated at initial temperature T_0=50 °C, at the beginning, due to any environment conditions. Liquefied gas, cooled to a storing temperature T_H=–50 °C is conveyed into the reservoir. The reservoir wall is exposed to temperature changes caused by cooling during the transient process. Cooling wall rate in the lower part of the vessel, i.e. the place where it is in contact with the liquid, is considerably higher compared to the cooling wall rate in the area above the liquid surface, where the gas phase is present. This occurs due to high coefficient of heat transfer from the wall to the liquid compared to the amount of the corresponding heat transfer coefficient from the wall to the gas phase i.e. $\alpha_1 >> \alpha_2$. Thus the temperature of the reservoir wall varies per meridian length. It can be expected that wall temperature bears high increase in the wall narrow band above the liquid surface. While the liquid level increases, this temperature increase rate decreases in the course of time. Its dependence on the level height will be determined in further consideration.

The spherical reservoir can be seen in Fig. 1. While reservoir loading the reached liquid level height in the fixed time moment $t=t_0$ is designated by a meridian angle $\vartheta=\vartheta_0$. The reservoir is supposed to be well thermally insulated. According to [3], the value of the Biot number for the part of the wall of the thin-walled vessel located above the liquid level is very small, $Bi<<1$. Therefore, it is reasonable to assume that the temperature in this wall is approximately uniformly distributed along the wall thickness at any time during the transient process, so the Lumped capacitance method can be used to determine its temperature.

In the area where the wall is located bellow the liquid level, the Biot number attains very high values. In this case the difference between the temperature of the inner surface of the wall and the liquid is small, $T_S \cong T_H$. As we are interested in the temperature of the inner surface only, the change of the temperature depending on the wall thickness will not be analized. Note that due to small wall thickness with good insulation and high thermal conductivity, the temperature of the steel wall becomes uniform shortly after sinking.

Initially, when the reservoir is still empty, the wall temperature is uniform in the whole superficial area of the vessel and equal to the environment temperature T_0. The reservoir loading rate by the cooled liquid was determined by the volume flowrate Q. Time t is required to pass so as to load the reservoir of R radius with liquid to the level given by the meridian angle ϑ:

$$t = \frac{V}{Q} = \frac{R^3 \cdot \pi}{3Q} \cdot \left(2 + 3 \cdot \cos\vartheta - \cos^3\vartheta\right) \tag{1}$$

where V is a part of the liquid-filled reservoir volume.

The reservoir wall was getting cool in the course of this period. Taking into account that the outer side of the reservoir is insulated, heat loss via insulation can be neglected. Due to high heat transfer coefficient, the wall temperature on the liquid-touched part is somewhat higher than that of the liquid itself. However, the wall gets cool considerably slower above the liquid surface. Its temperature in this area can be determined by using out heat balance equation. If a heat balance is considered for the wall part above the liquid surface, that has temperature T_S, that within the short time interval dt, gets cool by the rate dT_S, is expressed as:

$$\alpha \cdot F \cdot \left(T_S - T_H\right) \cdot \mathrm{d}t = -F \cdot h \cdot \rho \cdot c \cdot \mathrm{d}T_S \tag{2}$$

where α designates heat transfer coefficient, wall material density, c specific wall heat capacity, h wall thickness, F wall area above the liquid surface.

Integrating of equation (2) leads to the expression:

$$T_S - T_H = \left(T_0 - T_H\right) \cdot \mathrm{e}^{-\frac{\alpha}{c \cdot h \cdot \rho} \cdot t} \tag{3}$$

where t denotes loading time determined by the expression (1). Attained expression shows that the wall temperature above the liquid surface decreases in the course of time by the exponential act. Thus, according to [4], a time constant τ_1 can be introduced:

$$\tau_1 = \frac{c \cdot h \cdot \rho}{\alpha} \quad . \tag{4}$$

By the expression (1), at ϑ=0, time required for the reservoir to be fully loaded is as follows:

$$\tau_V = \frac{4 \cdot R^3 \cdot \pi}{3 \cdot Q} \, . \tag{5}$$

If a volume flowrate value Q determined by this expression is implied in the expression (1) and ratio $k = \tau_V / \tau_1$ is introduced we can get the expression for the reservoir loading time at the level given by the angle ϑ_0:

$$t_0 = \frac{k \cdot \tau_1}{4} \cdot \left(2 + 3 \cdot \cos\vartheta_0 - \cos^3\vartheta_0\right) \, . \tag{6}$$

Value of the k ratio can be calculated if geometrical physical parameters are known or determined experimentally by measuring time constant and time required for loading the

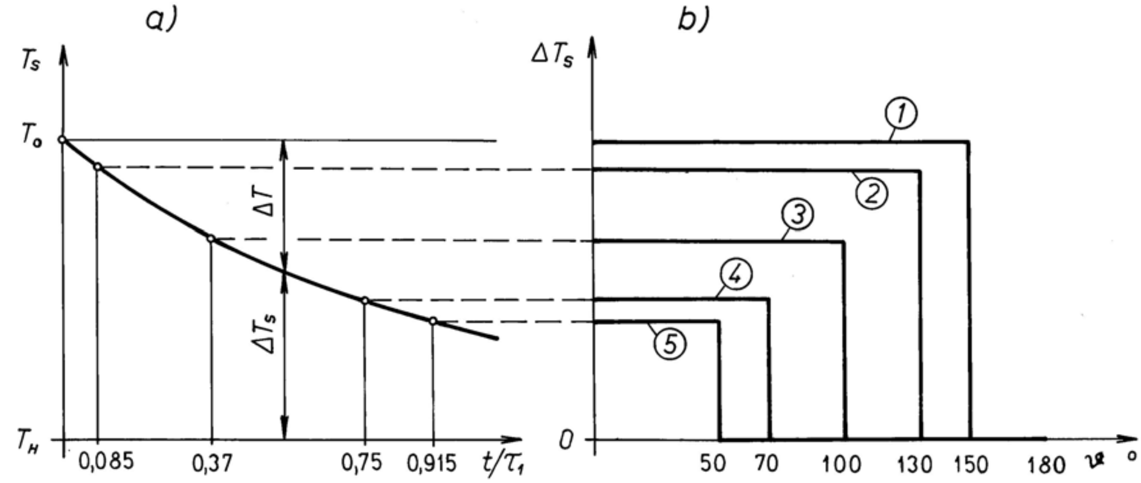

Figure 2 *a) Change of temperature differences in the course of time, b) Wall temperature increase at fixed times.*
Slika 2. *a) Promjena temeraturnih razlika tijekom vremena, b) Porast temperature u stijenci u fiksiranim trenucima.*

full reservoir τ_V.

After the reservoir loading time is calculated by the expression (6) to the given level, the wall temperature increase rate can be determined by the expression (3) at the reached liquid level in a certain moment:

$$\Delta T_{\mathrm{S}} = T_{\mathrm{S}} - T_{\mathrm{H}} = \left(T_0 - T_{\mathrm{H}}\right) \cdot \mathrm{e}^{-\frac{t}{\tau_1}} \tag{7}$$

as well as the sub-temperature of the reservoir wall part (compared to the referential temperature) located above liquid surface:

$$T_{\mathrm{S}} - T_0 = \left(T_{\mathrm{H}} - T_0\right) \cdot \left(1 - \mathrm{e}^{-\frac{t}{\tau_1}}\right). \tag{8}$$

Wall temperature and temperature differences defined by the expressions (7) and (8) are graphically presented in Fig 2.

Heat rejected by conducting through the reservoir wall into the meridian direction was neglected in the equation setting (2). This is justified if there is no temperature gradient in the wall, which is achieved for the part of the wall further from the liquid level. In the narrow area above the liquid level the wall experiences a prompt temperature increase in the meridian direction. This temperature gradient can be determined using the Fourier differential equation of heat diffusion through isotropic body, according to [5]. This equation gives:

$$\frac{\partial T_{\mathrm{S}}}{\partial t} = a \cdot \Delta T_{\mathrm{S}} + \frac{\dot{q}_{\mathrm{i}}}{c \cdot \rho} \tag{9}$$

where $\dot{q}_{\mathrm{i}}$ designates heat generation rate, Δ Laplace operator and $a = \lambda/(c \cdot \rho)$ thermal diffusivity.

Due to the spherical shape of the vessel wall the equation (9) is the most suitable for use in the spherical coordinate system. Since the temperature field is axi-symmetric and unchangeable per wall thickness, Laplace operator is reduced to the form:

$$\Delta = \frac{1}{R^2 \cdot \sin\vartheta} \cdot \frac{\partial}{\partial \vartheta} \left(\sin\vartheta \cdot \frac{\partial}{\partial \vartheta} \right). \tag{10}$$

Heat generation rate in the expression (9) presents heat supplied from the outside to the wall element per volume unit. It, in this example, refers to heat alternating on the internal wall:

$$\dot{q}_{\mathrm{i}} = \frac{\alpha}{h} \left(T_{\mathrm{H}} - T_{\mathrm{S}}\right). \tag{11}$$

By using the operator (10) and implementing expression (11) into (9), Fourier's equation is as follows:

$$\frac{\partial T_{\mathrm{S}}}{\partial t} = \frac{\lambda}{c \cdot \rho \cdot R^2} \cdot \left(\frac{\partial^2 T_{\mathrm{S}}}{\partial \vartheta^2} + \frac{1}{\tan\vartheta} \cdot \frac{\partial T_{\mathrm{S}}}{\partial \vartheta} \right) + \\ + \frac{\alpha}{c \cdot \rho \cdot h} \cdot \left(T_{\mathrm{H}} - T_{\mathrm{S}}\right). \tag{12}$$

Values of the temperature gradient $\partial T_{\mathrm{S}} / \partial \vartheta$ in the wall at the place of the liquid level are determined using the numerical methods of solving differential equation (12). These values are calculated in [3] and are being used further to approximate the function of the wall temperature increase.

3
Approximation of the wall temperature increase
Aproksimacija porasta temperature stjenke

Of all possible states in the course of the liquid level increase, a calculation has been conducted for five chosen levels. Values of the ratio t_0/τ_1 were determined by the expression (6) and substituted into expression (7) for fixed liquid level heights (given by the angle ϑ_0) and with expected value k=1. In this way pertaining temperature differences ΔT_{S}, representing temperature increase of the internal wall surface at fixed liquid level heights, were calculated. Their values are presented in Table 1. They were used for drawing approximate temperature distribution functions per reservoir meridian length and presented in Fig. 2b). Temperature distribution curves were designated by the circled numbers. They respond to timing order of setting up corresponding liquid level. For example, a curve designated no. 1 in Fig. 1 represents approximate temperature distribution of internal wall surface along the reservoir meridian at the moment when the liquid level reaches the height given by the angle ϑ_0=150°. Temperature increase presented as a discontinuity by this diagram occurs at a certain small meridian length in real situation. Due to the aforesaid these discontinuous functions are approximated by Fourier's orders in form:

$$T - T_0 = \mathrm{D}_0 + \mathrm{D}_1 \cdot \cos\vartheta + \mathrm{D}_2 \cdot \cos 2\vartheta + ... \qquad (13)$$

Fourier's orders were selected for approximation of the temperature distribution functions for mathematical reasons. Namely, such substitution enables obtaining explicit solution of corresponding differential equation for calculating temperature stresses. Coefficients of the Fourier's orders are calculated by means of the electronic calculator program. They can be seen in Table 1.

Figure 3 shows graphical representation of temperature distribution function approximation at the moment when the liquid level reaches the height of ϑ_0=150°.

Table 1 *Coefficients of Fourier's order for temperature distribution*
Tablica 1*. Koeficijenti Fourierovog reda za funkcije raspodjele temperature*

ϑ_0	50	70	100	130	150
ΔT_s	40 °C	48 °C	69 °C	92 °C	99 °C
D_0	–90,444	–83,466	–67,800	–42,759	–28,500
D_1	17,256	26,858	43,180	53,716	47,525
D_2	12,381	12,437	4,359	–19,470	–29,144
D_3	6,229	–1,023	–12,542	–7,200	9,128
D_4	0,801	–6,586	–3,710	12,300	4,209
D_5	–2,496	–4,180	5,700	–3,062	–8,095
D_6	–3,314	0,900	2,847	–4,925	5,310
D_7	–2,301	3,367	–2,673	4,468	–0,764
D_8	–0,593	1,994	–1,994	0,255	–2,038
D_{10}	0,786	–0,732	1,182	–2,670	2,294

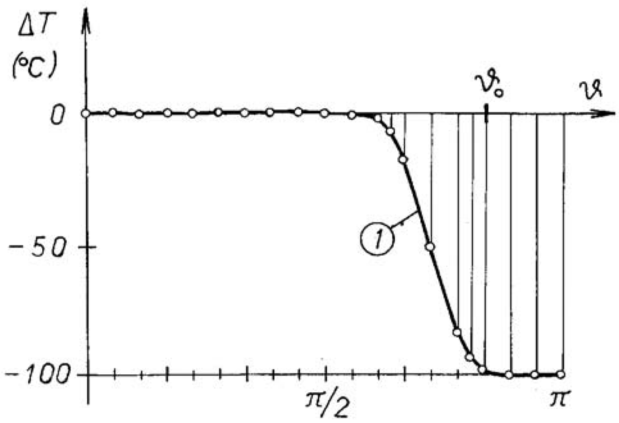

Figure 3 *Sub-temperature of the internal wall surface at liquid's level of* ϑ_0=150°
Slika 3*. Podtemperatura unutarnje površine stijenke pri razini kapljevine* ϑ_0=150°

4 Conclusion
Zaključak

Understanding of non-stationary temperature field in the reservoir wall allows temperature stresses calculation [6]. The method presented provides values of wall temperature increase in a parallel circle area with the liquid level. However, to analyse stress, it is not enough to know only the wall temperature increase ratio since temperature stresses depend on a temperature gradient. Temperature change function per reservoir meridian length, in reality, does not experience jump as in Fig. 2b, but it varies continuously as in Fig 3. Gradient of the function should be determined either by experimental methods, measuring wall temperature in the critical area or by numerical methods of solving differential equation (12). Taking into account the above mentioned it is enough to find out what temperature gradient values belong to certain temperature jumps i.e. determine dependence $\mathrm{d}(\Delta T)/\mathrm{d}\vartheta = f(\Delta T_s)$. These values do not significantly depend either on liquid type or level height they are measured at but only on the vessel wall thickness. This results from the assumption $\alpha_1 >> \alpha_2$. Furthermore, spherical reservoirs usually have high capacity (e.g. from 1000 to 5000 m^3), thus, while being loaded the liquid level increases relatively slowly. This is the reason why the wall temperature starts to rise just above the liquid level, as it is shown in Fig. 3.

Determined temperature distribution functions along the reservoir meridian were approximated by Fourier's orders of forms (13), whose coefficients can be seen in Table 1. The functions are used as a basis for further temperature stresses analysis that will be presented in the next paper.

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